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An improved analysis of deep inelastic neutrino data with the DGLAP equations at low and high x

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Abstract : The method of obtaining approximate solutions of DGLAP equations at low and high x reported earlier is re-examined and more general complete solutions are obtained. The formalism which enables one to separate out qualitatively the low x and high x regions explored in deep inelastic scattering is then tested with recent CCFR data with reasonable success.

Keywords : Structure functions, low and high x, DGLAP equation

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1. Introduction

DGLAP equations [1] are the basic tools to study the Q^2 evolutions of the structure functions. Thus even though BFKL [2],GLR [3] or more recent BK [4] equations are theoretically more appealing at low x, DGLAP equations are being used as the simplest perturbative tools, which are being relevant for the presently accessible $x - Q^2$ range of structure functions. In some of our earlier communications [5-8], it has been demonstrated that these equations can be transformed into a set of first order partial differential equations in x (the Bjorken variable) and $t = \log\left(Q^2/\Lambda^2\right)$, both at low and high x. As complete solutions of these equations with two differential variables in general need two boundary conditions, the vanishing of structure functions at $x \to 1$ was considered to be the additional boundary condition [7-9] besides the standard one of non-perturbative input at some low t

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$= t_0$. In this way, simple (t/t_0) behaviour at low x [5-7] and (t_0/t) behaviour at high x [8] for non-singlet structure functions were reported and tested with CCFR data [10, 11].

The aim of the present paper is to generalize the previous formalism, report complete solutions of approximated DGLAP equations both at low and high x and compare them with CCFR data.

In Section 2, we outline the formalism while Section 3 contains results and discussions. Section 4 ends with comments and conclusions.

2 Formalism

2.1 Low x limit of DGLAP equation:

The DGLAP equation [1] for non-singlet structure function in standard form is [12]

$$\frac{\partial F^{NS}(x,t)}{\partial t} = \frac{A_f}{t}\left[\{3+4\log(1-x)\}F^{NS}(x,t) + 2\int_x^1 \frac{dz}{1-z}\left\{\left(1+z^2\right)F^{NS}(x/z,t) - 2F^{NS}(x,t)\right\}\right]. \quad (1)$$

Here, $t = \log(Q^2/\Lambda^2)$ and $A_f = 4/(33-2N_f)$, N_f being the number of quark flavours. Introducing the variable $u = 1 - z$ and noting that [5-8]

$$\frac{x}{1-u} = x\sum_{k=0}^{\infty} u^k \,, \quad (2)$$

$F^{NS}(x/z,t)$ occurred in R.H.S. of eq. (1) can be expressed as:

$$F^{NS}(x/z,t) = F^{NS}(x,t) + \sum_{l=1}^{\infty}\left(x^l/l!\right)\left(\sum_{k=1}^{\infty} u^k\right)^l \frac{\partial^l F^{NS}(x,t)}{\partial x^l}. \quad (3)$$

For small x ($x \ll 1$), it is justified if higher order derivatives $\partial^l F^{NS}(x,t)/\partial x^l$ for $l > 1$ are neglected so that

$$F^{NS}(x/z,t) = F^{NS}(x,t) + x\sum_{k=1}^{\infty} u^k \frac{\partial F^{NS}(x,t)}{\partial x}. \quad (4)$$

Putting eq.(4) in eq.(1) and performing the u-integration, one gets

$$Q(t)\frac{\partial F^{NS}(x,t)}{\partial t} + P(x,t)\frac{\partial F^{NS}(x,t)}{\partial x} = R(x)F^{NS}(x,t) \quad (5)$$

with

$$P(x,t) = \frac{-A_f x[2\log(1/x) + (1-x^2)]}{t} = \frac{P_1(x)}{t}, \quad (6)$$

$$Q(t) = 1, \tag{7}$$

and

$$R(x,t) = \frac{A_f[3+4\log(1-x)+(x-1)(x+3)}{t} = \frac{R_1(x)}{t}. \tag{8}$$

Eq. (5) is frequently referred to as Lagrange's equation [13]. Its general solution is obtained by solving the following auxiliary systems of ordinary differential equations

$$\frac{dx}{P(x)} = \frac{dt}{Q(t)} = \frac{dF^{NS}(x,t)}{R(x)F^{NS}(x,t)}. \tag{9}$$

If

$$u(x,t) = C_1 \tag{10}$$

and

$$v(x,t) = C_2 \tag{11}$$

are two independent solutions of eq. (9), then the general solution of eq.(5) is

$$f(u,v) = 0, \tag{12}$$

where f is an arbitrary function of u and v.

Solving eq.(9),one obtains

$$u(x,t) = tX^{NS}(x), \tag{13}$$

and

$$v(x,t) = F^{NS}(x,t)Y^{NS}(x), \tag{14}$$

with

$$X^{NS}(x) = \exp\left[-\int \frac{dx}{P_1(x)}\right], \tag{15}$$

and

$$Y^{NS}(x) = \exp\left[-\int \frac{R_1(x)}{P_1(x)} dx\right]. \tag{16}$$

Explicit analytical forms of $X^{NS}(x)$ and $Y^{NS}(x)$ in the leading $1/x$ approximation are [7]

$$X^{NS}(x) \approx \exp\left[-\frac{1}{2}\log|\log x|\right], \tag{17}$$

and $$Y^{NS}(x) \approx 1. \tag{18}$$

The most general form of eq.(12) linear in F^{NS} (i.e. in v) is

$$v = \alpha u^{n(x,t)} + \beta , \tag{19}$$

where $n(x, t)$ is any real function of x and t, and β ,α are two arbitrary constants. Reality condition on structure functions forbids $n(x, t)$ being complex. $n(x,t) = 1$ suggested in earlier communications [5-8] is just a particular case of (19).

In order to proceed further, we use the following two physically plausible boundary conditions:

$$F^{NS}(x,t) = F^{NS}(x,t_o) . \tag{20}$$

for some low $t = t_0$ and

$$F^{NS}(1,t) = 0 . \tag{21}$$

for any t [7-9], consistent with quark counting rules [14,15].

Using the boundary conditions of eq.(20) and eq.(21) in eq. (19), we get

$$F^{NS}(x,t_0)Y^{NS}(x) = \alpha\left[t_o X^{NS}(x)\right]^{n(x,t)} + \beta \tag{22}$$

and

$$0 = \alpha\left[tX^{NS}(1)\right]^{n(x,t)} + \beta \tag{23}$$

which leads to

$$F^{NS}(x,t) = F^{NS}(x,t_o)(t/t_o)^{n(x,t)} \frac{[X^{NS}(x)]^{n(x,t)} - [X^{NS}(1)]^{n(x,t)}}{[X^{NS}(x)]^{n(x,t)} - (t/t_o)^{n(x,t)}[X^{NS}(1)]^{n(x,t)}} . \tag{24}$$

As from eq.(17),

$$X^{NS}(1) \approx 0 . \tag{25}$$

Equation (24) yields for $n(x, t) > 0$,

$$F^{NS}(x,t) = F^{NS}(x,t_o)(t/t_o)^{n(x,t)} , \qquad n(x,t) > 0 . \tag{26}$$

On the other hand for $n(x, t) < 0$, numerically, the R.H.S of eq. (24) will involve inverse of $X^{NS}(1)$ which is singular and ill-defined and hence excluded on physical grounds. Thus at low x, eq.(26) is the physically plausible general solution of Lagrange's equation (5).

This is to be compared with the particular solution at low x reported earlier [5-7]

$$F^{NS}(x,t) = F^{NS}(x,t_o)(t/t_o)\,. \tag{27}$$

If further, the constants β and α in equation (19) satisfy an additional relation [16]

$$\beta = \alpha^m (m \neq 1)\,, \tag{28}$$

eq. (26) will have the form :

$$F^{NS}(x,t) = F^{NS}(x,t_o)(t/t_o)^{\frac{m\,n(x,t)}{m-1}}\,. \tag{29}$$

Eq.(26) and eq.(29) coincide for large m so that $m/(m-1) \approx 1$. For finite m, they look different. The factor $m/(m-1) \approx 1$ can be absorbed in the redefinition of $n(x, t)$ as

$$n'(x,t) = \frac{m}{m-1}\,n(x,t) \tag{30}$$

with the condition $n'(x,t) > 0$.

Let us now discuss about the possible compatibility of eq.(26) with known asymptotics of double leading logarithmic approximation [17]:

$$F^{NS}_{DLA} \approx \exp\left[A(Q^2)t\log(1/x)\right]^{1/2}, \tag{31}$$

where for three colours ($N_e = 3$) and four flavours ($N_f = 4$),

$$A(Q^2) \approx \frac{8}{25}\log t\,. \tag{32}$$

Compatibility of eq. (31) with eq. (26) yields explicit x and t-dependence for $n(x, t)$

$$n(x,t) = \left\{\frac{8}{25}t\frac{\ln(1/x)}{\ln t}\right\}^{1/2}. \tag{33}$$

This is to be compared with the corresponding compatibility condition for the particular solution eq. (27)

$$\frac{\log t}{t} = \frac{8}{25}\log(1/x) \tag{34}$$

obtained earlier [7], valid specifically for $x \geq 0.044$ for any $t > t_0$.

It is to be noted that the condition $n(x, t) > 0$ in eq.(26) or equivalently $n'(x,t) > 0$ in eq.(30) at small x is essentially because of the behaviour of $X^{NS}(x)$ at $x = 1$ [eq.(25)] which occurs in eq.(24) due to the boundary condition at large x [eq.(21)].This feature indicates the implicit correlations of partons at different x ranges as noted in a recent global QCD analysis [18].

2.2 High x limit of DGLAP equations:

In order to study the high x limit of DGLAP equation (1), let us reconsider eq.(3) which contains two infinite series, one in x and the other in u. As $x \to 1, u \to 0$, so it is reasonable to write

$$F^{NS}(x/z,t) = F^{NS}(x,t) + \sum_{l=1}^{\infty}\left(x^l/l!\right)u^l \frac{\partial^l F^{NS}(x,t)}{\partial x^l}. \tag{35}$$

In eq.(35), the small expansion parameter is xu rather than x. Assuming that the higher order derivatives of non-singlet structure functions are non-singular as $x \to 1$, eq. (35) can be further reduced to [8]

$$F^{NS}(x/z,t) = F^{NS}(x,t) + xu\frac{\partial F^{NS}(x,t)}{\partial x}. \tag{36}$$

to be compared with eq. (4).

Using eq.(36) in eq.(1) and performing the u -integration, we get an equation similar to eq. (5), except for the replacement

$$P(x,t) \to P'(x,t) = \frac{2}{3}\frac{A_f x(x-1)(x^2+x+4)}{t} = \frac{P_1'(x)}{t} \tag{37}$$

i.e.

$$Q(t)\frac{\partial F^{NS}(x,t)}{\partial t} + P'(x,t)\frac{\partial F^{NS}(x,t)}{\partial x} = R(x)F^{NS}(x,t). \tag{38}$$

Its general solution is again of the type, equation (12), *viz*

$$h(u',v') = 0, \tag{39}$$

where h is another arbitrary function of u' and v' obtained by Lagrange's method of solutions of eq. (38):

$$u'(x,t) = tX'^{NS}(x), \tag{40}$$

$$v'(x,t) = F^{NS}(x,t)Y'^{NS}(x), \tag{41}$$

with

$$X'^{NS}(x) = \exp\left[-\int\frac{dx}{P_1'(x)}\right] \tag{42}$$

and
$$Y'^{NS}(x) = \exp\left[-\int \frac{dxR(x)}{P_1'(x)}\right] \tag{43}$$

to be compared with eq. (15) and eq.(16).

Explicit analytical form of $X'^{NS}(x)$ is [8]

$$X'^{NS}(x) = \exp\frac{1}{A_f}\left(\frac{\sqrt{3/5}}{8}\arctan\frac{1+2x}{\sqrt{15}} - \frac{\log(1-x)}{4} + \frac{3\log(x)}{4} - \frac{\log(x^2+x+4)}{16}\right. . \tag{44}$$

which is singular as $x \to 1$ due to the $\log(1-x)$ factor in it.

The most general form of eq.(39) linear in structure function (*i.e.* in v') is :

$$v' = \alpha' u'^{p(x,t)} + \beta', \tag{45}$$

where $p(x, t)$ is another function of x and t similar to $n(x, t)$ in eq.(19).

Using the boundary conditions eq.(20) and eq.(21) in eq. (45) to determine α' and β', we get

$$F^{NS}(x,t) = F^{NS}(x,t_o)(t/t_o)^{p(x,t)} \frac{[X'^{NS}(x)]^{p(x,t)} - [X'^{NS}(1)]^{p(x,t)}}{[X'^{NS}(x)]^{p(x,t)} - (t/t_o)^{p(x,t)}[X'^{NS}(1)]^{p(x,t)}} . \tag{46}$$

If $p(x, t)$ is numerically positive ($p(x, t) > 0$), R.H.S. of eq.(46) is ill-defined due to the singular nature of $X'^{NS}(1)$ defined in eq.(44). If on the other hand, if $p(x, t)$ is numerically negative ($p(x, t) > 0$), R.H.S. of eq.(46) involves $\left[X'^{NS}(1)\right]^{-1}$ which vanishes identically. This results in the general solution at high x to be

$$F^{NS}(x,t) = F^{NS}(x,t_o)(t/t_o)^{p(x,t)}. \qquad (p(x, t) > 0) \tag{47}$$

This is to be compared with the particular solution reported earlier [8]:

$$F^{NS}(x,t) = F^{NS}(x,t_o)(t_o/t). \tag{48}$$

As noted in eq.(30), additional adhoc relation like (28) does not yield any new testable prediction even in high x limit.

3. Results and discussion

Let us make a reanalysis of recent CCFR data[10,11] and see how they conform to general formalism in Section 2. In order to do it, we redefine eq.(26) and eq. (47) as

$$F^{expt}(x,t) = F(x,t_0)(t/t_0)^{H_{expt}(x,t)}, \tag{49}$$

such that for $x \to 0$, $H_{expt}(x,t) \to n(x,t)$, and for $x \to 1$, $H_{expt}(x,t) \to p(x,t)$. resulting in

$$H_{\exp t}(x,t)=\frac{\log F^{\exp t}(x,t)-\log F(x,t_0)}{\log(t/t_0)}. \tag{50}$$

For comparison, we also take the DLA limit [7,17], eq. (33) for low x, while for high x, the result obtained in Ref. [19]

$$p(x,t)=-\frac{4}{25}[3+4\log(1-x)] \tag{51}$$

for $n_f=4$.

For numerical analysis, we take the non-singlet structure function $xF_3(x,t)$ in leading order [20] as

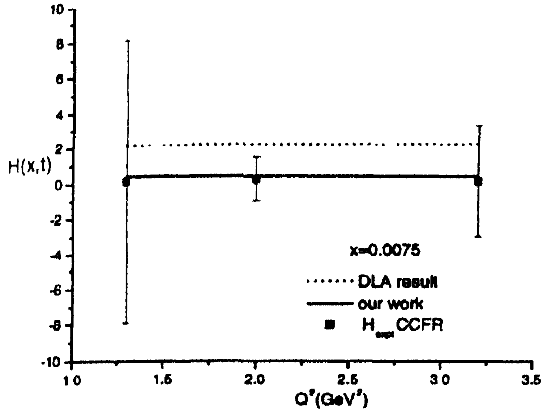
$$xF_3(x,t)=-x(\bar{u}+\bar{d})+x(d+u)|v_{ud}|^2+2sx|v_{us}|^2, \tag{52}$$

where $u=u(x,t)$ *etc.* and v_{ud}, v_{us}, *etc.* are the relevant CKM matrix elements [21].

For inputs, we use MRS 2001 LO parton distributions [22] at $Q_0^2=1\,\mathrm{GeV}^2$ and with Λ_{LO}=220 MeV.

In figures 1 (a-r), we plot $H_{\exp t}$ *vs.* for $Q^2 x$ = 0.0075, 0.0125, 0.0175, 0.0250, 0.0350, 0.0500, 0.0700, 0.0900, 0.1100, 0.1400, 0.1800, 0.2250, 0.2750, 0.3500, 0.4500, 0.5500, 0.6500, 0.7500, respectively.

From the variation of $H_{\exp t}$ with Q^2 for different values of x, it is quite evident that it does not coincide with the values, $H_{\exp t}$ = 1 for low x and $H_{\exp t}$ = − 1 for high x corresponding to particular solutions of eqs. (27) and (48), respectively. The deviation of $H_{\exp t}$ from ± 1, as revealed in our analysis, clearly points towards the justification of our general solutions of eq. (26) and eq. (47), respectively. Numerically for low x, most of data for $H_{\exp t}$ is found to be positive and is confined in the range 0.018< $H_{\exp t}$ <1.58, while for high x, it is found to be negative in the range 0.070< $|H_{\exp t}|$ <1.77 barring one data point. However in the following few (x, Q^2) values, $H_{\exp t}$ for low x turn out to be negative: (0.0125, 3.2), (0.0175, 1.3), (0.0250, 1.3), (0.0500, 1.3), (0.0700, 2), (0.0900, 3.2), (0.0900, 50.1), (0.1100, 3.2), (0.1400, 3.2). Again on the high x side, $H_{\exp t}$ comes out to be positive at the data point (.1800, 5). From the point of view of the present formalism, these few points can be termed as anomalous. Barring these few points, we can divide the x-range as explored in CCFR experiment [11] into two regions- low $x(.0075\le x\le .1400)$ and high $x(.1800\le x\le .7500)$. The low and high x regions have respectively $H_{\exp t}>0$ and $H_{\exp t}<0$ numerically as per our prediction of eq.(26) and eq.(47). The first ten figures of Figure (1) (a-j) represent the low regime and the next eight figures of Figure 1 (k-r) represent the high x regime.



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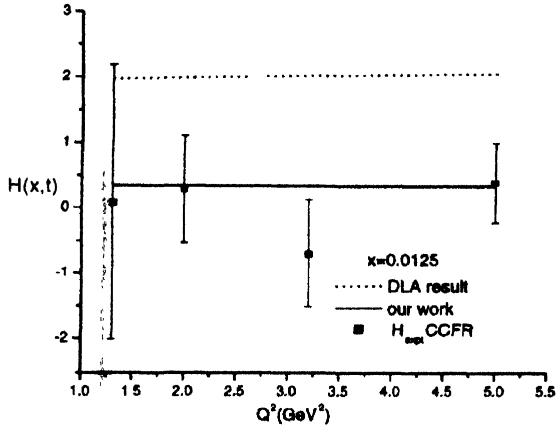

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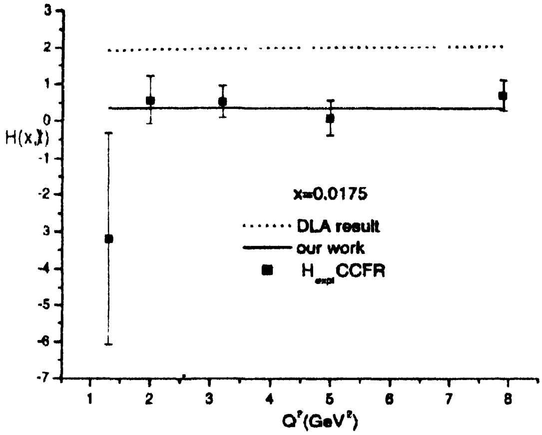

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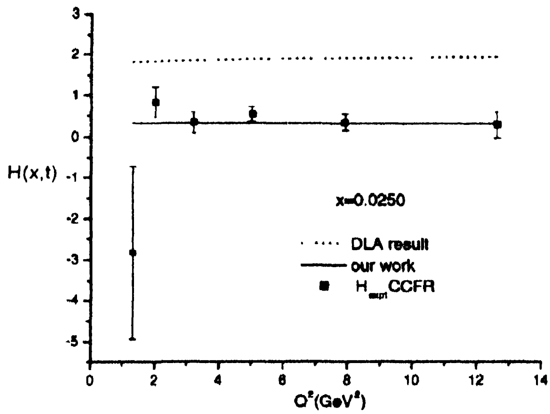

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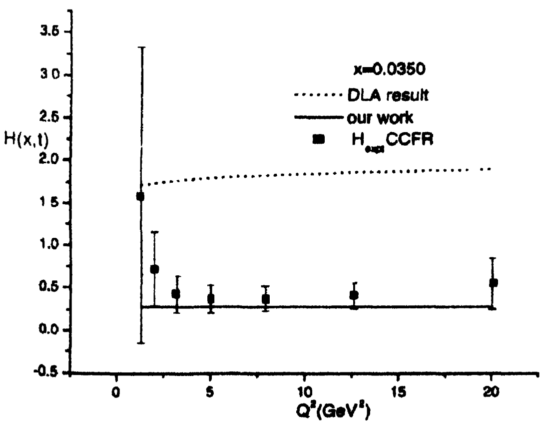

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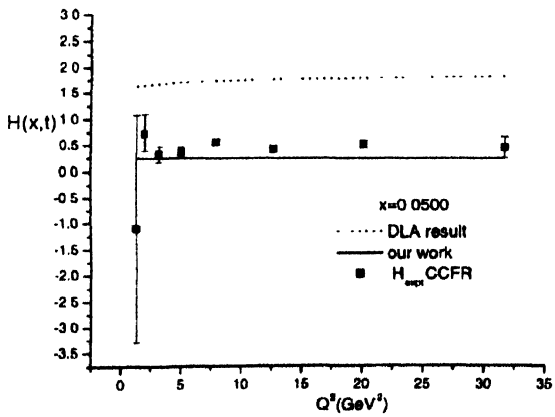

Figure 1(f)

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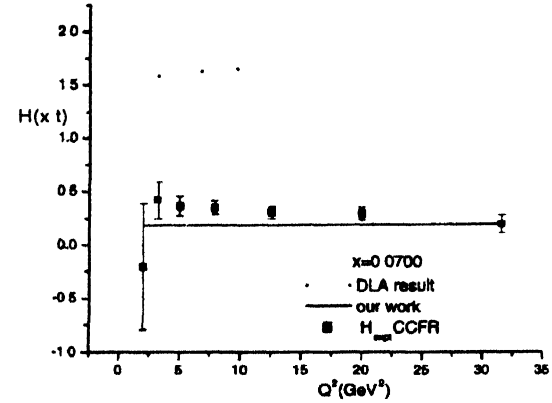

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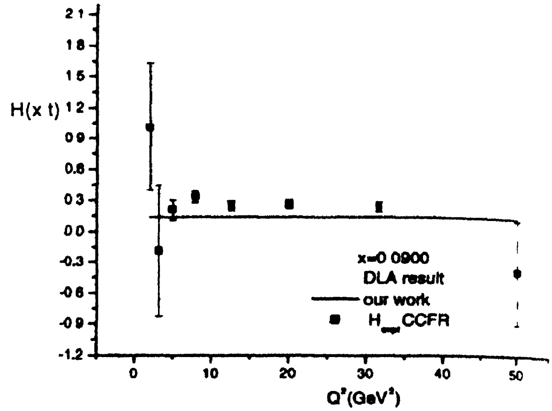

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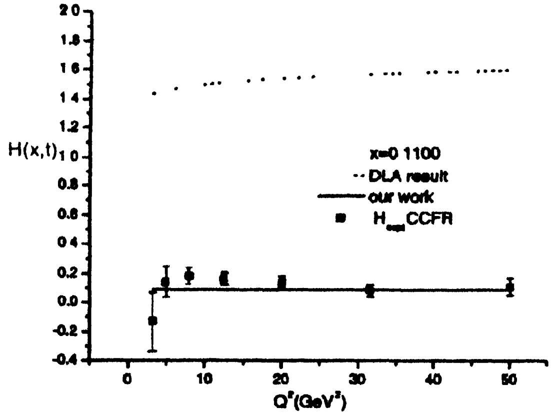
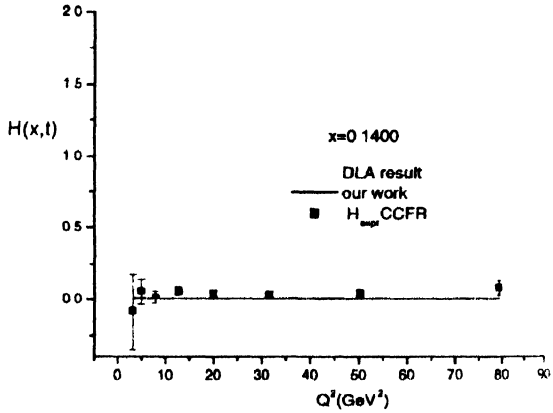


Figure 1(i)



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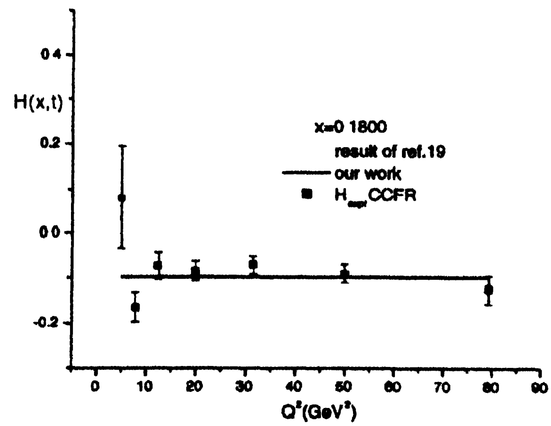

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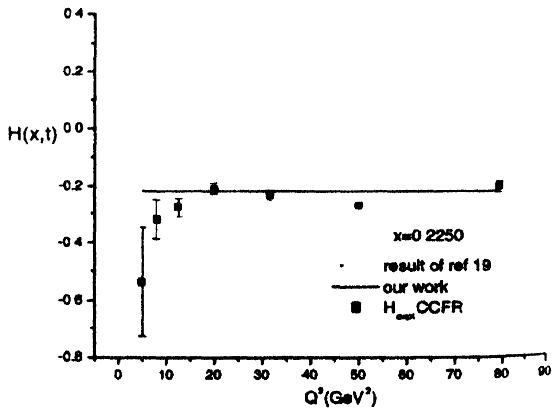

Figure 1(l)

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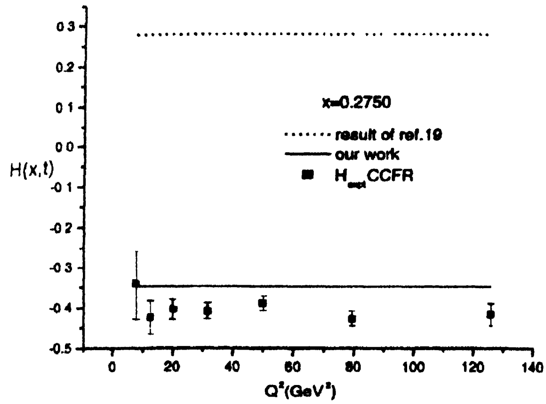

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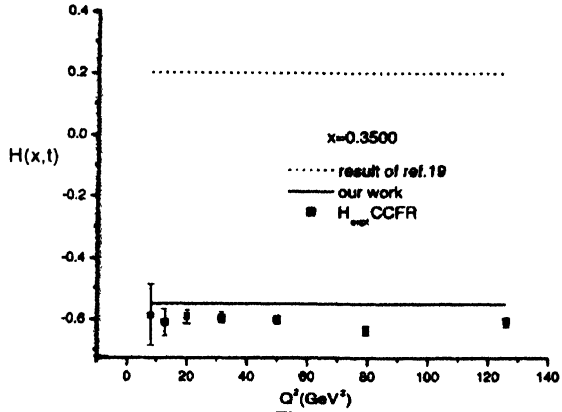

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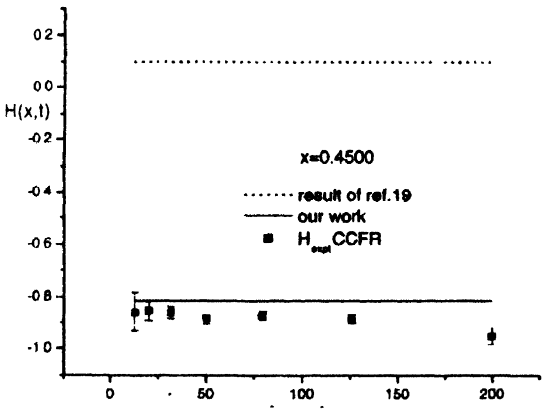

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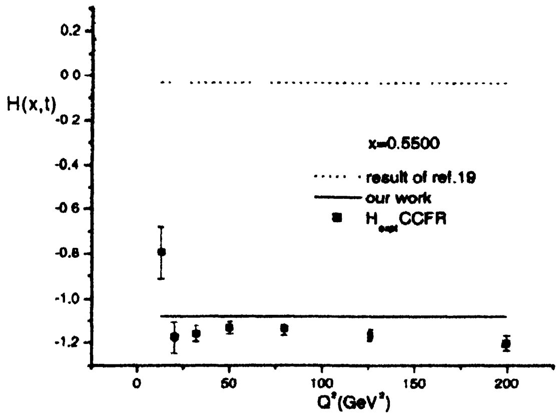

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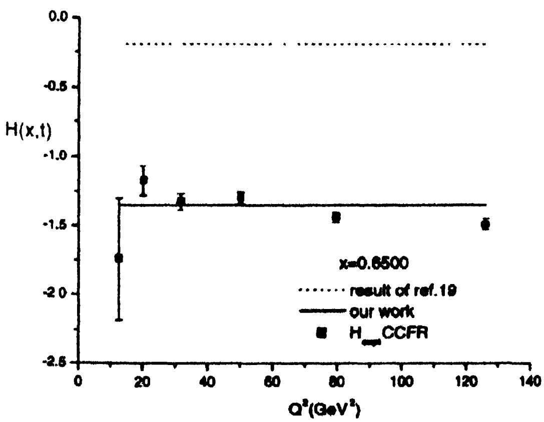

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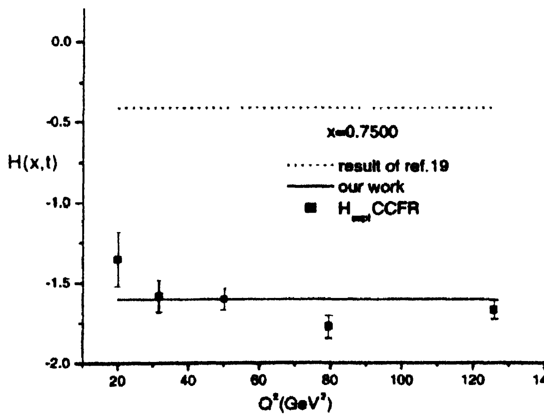

Figure 1(r)

Figure 1. (a-r): H_{expt} *vs.* Q^2 (GeV^2) for x = 0.0075, 0.0125, 0.0175, 0.0250, 0.0350, 0.0500, 0.0700, 0.0900, 0.1100, 0.1400, 0.1800, 0.2250, 0.2750, 0.3500, 0.4500, 0.5500, 0.6500, 0.7500

For comparison, in the same figures, we also plot (dotted curve) the values of $n(x, t)$ from eq.(33) [Figure 1 (a-j)] and $p(x,t)$ from eq.(51) [Figure 1 (k-r)] for all the representative values of x.

We also draw solid line based on empirical formula (62).

For comparison in the same figures, we also plot $n(x, t)$ vs Q^2 (Dotted curve) from equations (33) and $p(x, t)$ vs Q^2 from eq.(51) (dotted curve) in the low [Figure 1 (a-j)] and high x [Figure 1 (k-r)] regimes respectively. None of them conforms to the data well.

Let us now suggest a plausible form of $H(x, t)$ compatible with eq. (26) and eq. (47)

$$H(x,t) = -\sum_i \alpha_i(x,t)x^i + \sum_j \beta_j(x,t)(1-x)^j, \tag{53}$$

where the functions $\alpha_i(x,t)$ and $\beta_j(x,t)$ are positive in the entire x, t plane $0< x <1$, $t_0 < t < \infty$ such that

$$\lim_{(1-x)\to 0} \alpha_i(x,t) = \alpha_i(1,t), \tag{54}$$

$$\lim_{x\to 0} \beta_j(x,t) = \beta_j(0,t). \tag{55}$$

Making Taylor's expansion of $\alpha_i(x,t)$ and $\beta_j(x,t)$ around $(1-x)\to 0$ and $x\to 0$ and keeping the first three terms,

$$\alpha_i(x,t) = \alpha_i(1,t) + (1-x)\ \alpha_i'(1,t) + \frac{(1-x)^2}{2!}\ \alpha_i''(1,t), \tag{56}$$

$$\beta_j(x,t) = \beta_j(0,t) + x\beta_j'(0,t) + \frac{x^2}{2!}\beta_j''(0,t). \tag{57}$$

Putting eq.(56) and eq.(57) in (53), one gets the most general form of $H(x, t)$ which contains the correct limiting behaviour $n(x, t) > 0$ and $p(x, t) > 0$ as required by the formalism proposed in Section 2 :

$$H(x,t) = \sum \alpha_i(1,t) + (1-x)\ \alpha_i'(1,t) + \frac{(1-x)^2}{2!}\alpha_i''(1,t) \quad x^i$$

$$\sum_j \beta_j(0,t) + x\beta_j'(0,t) + \frac{x^2}{2!}\beta_j''(0,t)\ |(1-x)^j. \tag{58}$$

In order to make a phenomenological analysis of CCFR data [10,11] with the minimum number of adjustable parameters, we assume $H(x, t)$ to be of simpler form

$$H(x,t) = -\alpha(1,t)\,x + \beta(0,t)(1-x) \tag{59}$$

which still satisfies the correct low and high x behaviour of eq.(26) and eq.(47). In general, x and Q^2 dependence of the exponent $H(x, t)$ defined in eq. (59) is not factorizible since

$\alpha(1,t)$ and $\beta(0,t)$ may have different t-dependences. As the simplest possible model of $H(x, t)$, in the following, we assume $\alpha(1,t)$ and $\beta(0,t)$ have identical t-dependences :

$$\alpha(1,t) = \alpha\, h(t) \tag{60}$$

and

$$\beta(0,t) = \beta\, h(t)\,, \tag{61}$$

so that $H(x, t)$ has factorizible x and t dependences

$$H(x,t) = h(t)[-\alpha\, x + \beta(1-x)]\,. \tag{62}$$

The motivation of such additional assumption on the exponent is its inherent simplicity for phenomenological study with minimum number of adjustable parameters.

We make analysis of CCFR data [10,11] and obtain $\alpha \approx 5.976$ and $\beta \approx 0.9961$ suggesting negligible t-dependence of $h(t)$ as is evident from the figures 1(a-r) (Solid lines) for $H(x, t)$.

In Table 1, we record the numerical values of $h(t)$ for various Q^2,

Table 1. Numerical values of $h(t)$ for various Q^2.

Q^2 (GeV^2)	$h(t)$
12.6	0.3611
20	0.3657
31.6	0.3706
50.1	0.3778
79.4	0.4035
125.9	0.4057

which is also displayed graphically in Figure 2. $h(t)$ is seen to have a mild rise with t. In Table 2, we record χ^2 with $h(t)$ = 0.3611, 0.3804 and 0.4057 for ten representative (x, Q^2) points: (0.0075, 1.3), (0.0125, 5), (0.0175, 7.9), (0.0250, 12.6), (0.0350, 20) (0.050, 31.6), (0.1100, 50.1), (0.1800, 79.4), (0.2750, 125.9) and 0.4500, 199.5), respectively.

Table 2. Values of χ^2 with $h(t)$.

$h(t)$	χ^2	χ^2 / d.f.
0.3611	0.1331	0.0148
0.3804	0.1225	0.0136
0.4057	0.1109	0.0123

It shows that there is no significant difference of χ^2 among the three estimated values suggesting the correctness of negligible Q^2 dependence of $h(t)$.

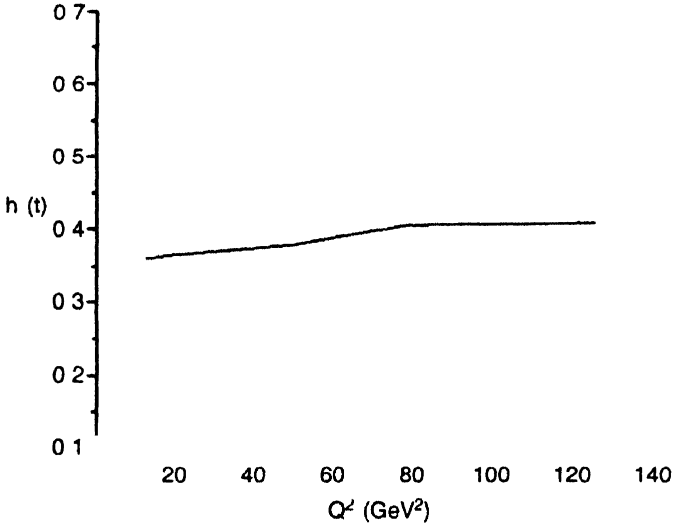

Figure 2. Plot of $h(t)$ vs Q^2 (GeV^2)

In Figures 3(a-f), we plot $H(x, t)$ *vs.* x for representative values of Q^2 = 12 6, 20, 31 6 50 1, 79.4 and 125 9, respectively It indicates that the empirical formula (62) agrees well with the CCFR data points The few low Q^2 data ($Q^2 = 1.3 - 7.9\ GeV^2$) however, cannot be accommodated with such a simple form of $H(x, t)$ suggesting its validity only for $Q^2 > 7.9\ GeV^2$

4. Comments and Conclusion

In this paper, we have generalized the method of obtaining approximate solutions of DGLAP equation at low and high x reported earlier[7,8] and analyzed CCFR data with it. A plausible theoretical form of the exponent $H(x, t)$ is then suggested and a simple empirical form of (62) is found to accommodate most of the CCFR data satisfactorily

For completeness, we also note that since usually the exponents of (t / t_0) in structure functions are expected to depend on QCD β functions and anomalous dimensions [23] the function $H(x, t)$ occurred in our analysis should be related to them as well However, to derive an explicit relation between the two, is beyond the scope of the present analysis.

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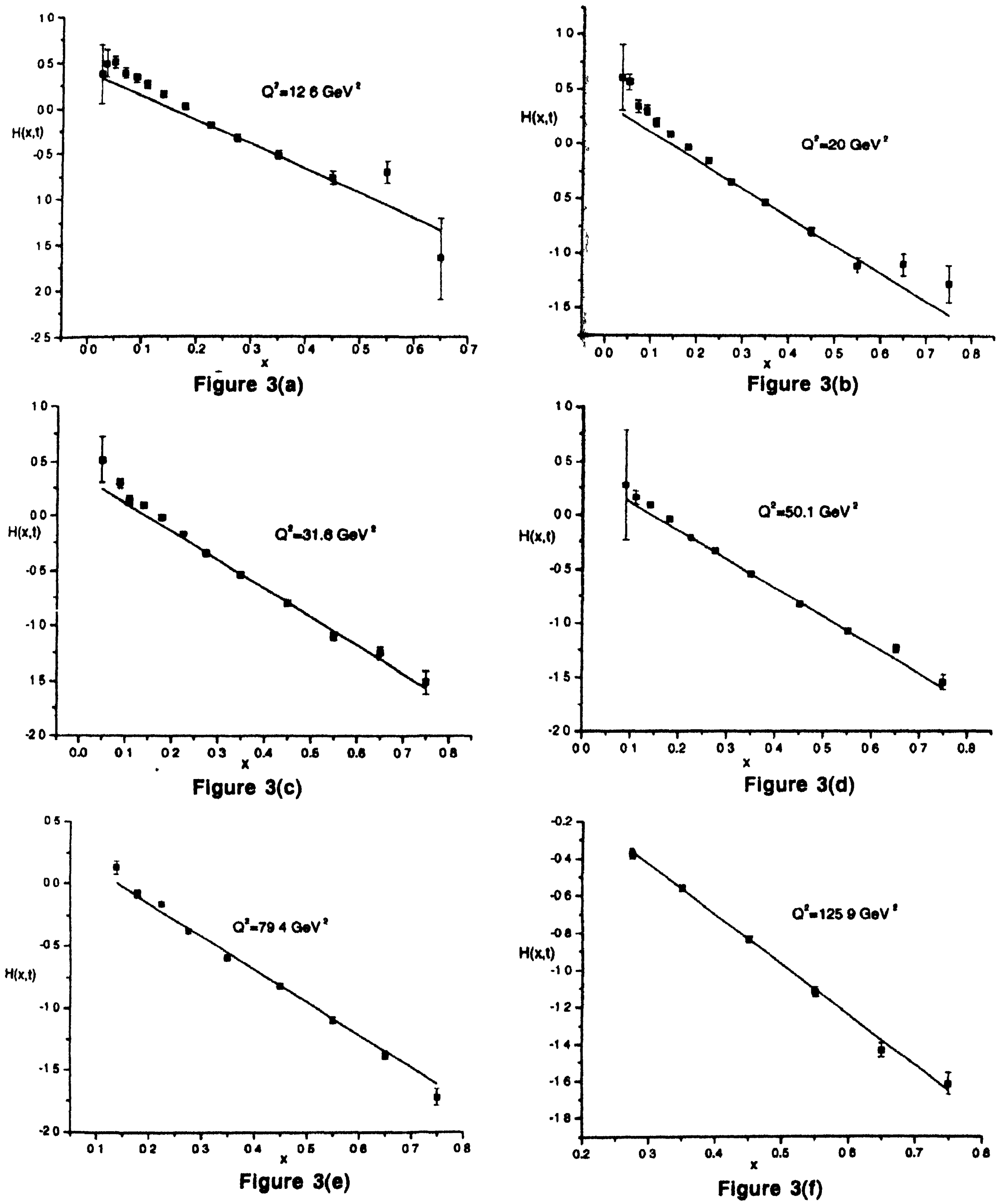

Figure. 3 (a-f). Plots $H(x, t)$ vs. x for representative values of Q^2 = 12.6, 20, 31.6, 50 1, 79.4 and 125.9, respectively. For comparison, we draw solid line based on the empirical formula (62).

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